Abstract

The Dirac Equation is one of the most beautiful mathematics expressions in the history. It successfully merges special relativity with quantum mechanics, predicts the existence of antimatter and paves the way to quantum field theory. However, many predictions such black-holes in Bose-Einstein condensates and Zitterbewegung are not possible to be observed directly due to the technical constraints. In this report, we simulate the Dirac equation in 1+1 dimensions by trapping a single $^{40}Ca+$ ion in a linear Paul trap with laser coupling of axial trapping frequency 2π x 1.36 MHz and radial trapping frequency 2π x 3 MHz. The Dirac equation is simulated by applying Dirac Hamiltonian H_D for varying amounts of time and for different particle masses. Measurements in position space are carried out by mapping the observable of interest onto the ion's internal state σ. The data collected well fits the theoretical prediction of Zitterbewegung of the particle. The experiment implemented a proof-of-principle quantum optical simulation of a tunable relativistic quantum mechanical system and showed a possibility for more complicated quantum simulations.

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Words: 173

Summary

The letter Quantum Simulation of the Dirac Equation, discussed the experiment of using a single $^{40}Ca+$ ion to simulate the 1+1 dimensional Dirac equation and investigate Zitterbewegung.

Dirac equation is famous for merging special relativity with quantum mechanics and predicting the existence of antimatter. It is also famous for its counterintuitive prediction of Zitterbewegung, Klein's Paradox and other quantum phenomenon. Zitterbewegung refers to the theoretical rapid fluctuation of a free particle caused by the interference of the positive and negative energy spinor states. The magnitude of the frequency and wavelength are beyond today's technological availability to be directly measured.

In the letter, the scientists aimed to simulate the Zitterbewegung using a controllable laboratory system that underlies the same mathematical model. The Dirac equation can be simulated using laser coupling to the three vibrational eigenmodes and the internal states of a single trapped ion.

Mathematically, the evolution of a massive particle is described by $\hat{x}(t) = \hat{x}(0) + \hat{p}c^2 H_D^{-1} t + i\hat{\xi}(e^{2iH_D t/\hbar} - 1)$, where $\hat{\xi} = (1/2)\hbar c(\sigma_x - \hat{p}c^2 H_D^{-1} t) H_D^{-1}$. By controlling the parameter $\hat{\Omega}$ and Ω (adjusting the bichromatic light field), only the third term of the equation, which may describe the Zitterbewegung, varies.

In the experiment, a single $^{40}Ca+$ ion was trapped in a linear Paul trap with laser coupling of axial trapping frequency $2\pi \times 1.36$ MHz and radial trapping frequency $2\pi \times 3$ MHz.. A single trapped ion allows exceptional control of experimental parameters, and initialization and read-out can be achieved with high fidelity. The Dirac equation was simulated by applying Dirac Hamiltonian H_D for varying amounts of time and for different particle masses. H_D was simplified to $H_D = 2\eta\Delta\hat{\Omega}\sigma_x\hat{p} + \hbar\Omega\sigma_z$ in this 1+1 dimension case, where $\Delta = \sqrt{\hbar / 2\tilde{m}\omega_{ax}}$ was the size of the ground-state wave function with $\tilde{m}$ the mass of $^{40}Ca+$ ion; η was the Lamb-Dicke parameter; and $\hat{p} = i\hbar(a^\dagger - a)/2\Delta$ was the momentum operator, with a and $a^\dagger$ the operator for the motional states along the axial direction.

The relationship between the expectation value of the position operator of the harmonic oscillator $<\hat{x}(t)>$ and the time was what the scientists was looking for. To measure $<\hat{x}(t)>$ for a motional state of ρ_m, they first prepared the ion's internal state in an eigenstate of σ_y . Then they applied a unitary transformation $U_{(\tau)}$ that mapped information about ρ_m onto the internal states. At last, they recorded the changing excitation as a function of the probe time τ, by measuring fluorescence.

The expectation values versus the evolution time graphs obtained showed that the frequency increased while the amplitude decreased as the mass of the particles increased. It confirmed that Zitterbewegung decreased in both limits, as theoretically expected. Further applying the tools to set the initial state of the simulated particle and contrasting the result with the result obtained from reconstructing probability distributions, all at t=0,75 and 150μs, it clearly showed that Zitterbewegung appeared only when there was interference of positive and negative energy parts of the state.

The experiment had two significant impacts: 1), It showed approximately that long-argued Zitterbewegung to be truly existing. 2), It implemented a proof-of-principle quantum optical simulation of a tunable relativistic quantum mechanical system. It showed great potential and enormous possibilities that more complex quantum systems and theories may be well simulated in the near future.

The conclusion drawn is fairly reasonable. However, several approximations and assumptions need to be taken into consideration. The most significant one is that particles, such as electrons, having much less mass may not behave as a $^{40}Ca+$ ion. To experimentally show Zitterbewegung to be true, some huge advancement in technology needs to be achieved.

This letter is really valuable. In order to understand the article, I self-studied the mathematics about the Dirac equation and gained an insight of how it was established. I also understood how ‘experimental simulation’ worked and helped me gain a deeper insight into another article which talked about the simulation of black holes by using sonic systems.

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WORDS: 618